On the Specific Resistance of Mercury

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XIV. *On the Specific Resistance of Mercury.*

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[Plate 19.]

Of late years several determinations of the electrical resistance of mercury have been made, and the differences between the results arrived at have been greater than would be expected at first sight from the nature of the observations involved. The results of the experiments have been expressed either in terms of the *ohm* (10^9 absolute C.G.S. units) or of the B.A. unit, which, according to the determinations of Lord Rayleigh and one of the authors of this paper (R. T. G.), is equal to ·98667 ohm.

In the case of Lord Rayleigh's observations, a direct comparison was made between the mercury unit and the original B.A. standards. Other observers have constructed copies of their mercury resistances in German-silver wire, which have been compared with the B.A. standards at the Cavendish Laboratory by one of us, or have compared their tubes directly with copies in platinum-silver wire of the B.A. units which have been sent from Cambridge after careful testing. The result of these various comparisons of recent years is as follows, and may conveniently be put in tabular form, giving the value in B.A. units of the resistance of a column of mercury 1 metre long, 1 square millimetre in cross section, at 0° Centigrade.

This is done in Table I., which also gives the value as found by various observers of the resistance of 1 ohm expressed in centimetres of mercury at 0° C.

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TABLE I.

Observer.	Date.	Value of 1 metre of mercury in B.A. U.	Value of Ohm in centimetre of mercury.
Lord RAYLEIGH and Mrs. SIDGWICK*	1883	·95412	106·23
MASCART, NERVILLE, and BENOIT†	1884	·95374	106·33
STRECKER‡	1885	·95334	—
L. LORENTZ §	1885	·95388	105·93
ROWLAND ‖	1887	·95349	106·32
KOHLRAUSCH ¶	1887	·95331	106·32
GLAZEBROOK and FITZPATRICK	1888	·95352	106·29
WUILLEUMIER**	1888	·95355	106·27

Since the original standards of the British Association are at Cambridge, it is possible to compare there the mercury unit and the B. A. unit directly; and, on hearing the results of Professor ROWLAND's careful investigations, communicated to the British Association at Manchester, it was thought advisable by several members of the Electrical Standards Committee to repeat the experiments at the Cavendish Laboratory. This seemed the more desirable, as the number expressing the B.A. unit in terms of the ohm is given by Professor ROWLAND as ·98644; and this agrees much more closely with the results of the similar observations at Cambridge, viz., ·98667, than do the corresponding values of the mercury unit in terms of the B.A. unit, viz., ·95349 (ROWLAND), and ·95412 (RAYLEIGH).

It may be useful to state clearly what is meant by the B.A. unit. In 1864 Messrs. MATTHIESSEN and HOCKIN constructed a number of coils of various materials to represent at certain specified temperatures resistances of 10^9 C.G.S. units of resistance as determined by the Electrical Standards Committee.

Eight of these coils (two being of platinum-iridium, two of platinum, three of platinum-silver, and one of gold-silver) have been retained in the possession of the Committee, while copies have been distributed to other electricians.

The temperatures at which six of these coils are equal to each other, and to one B.A. unit, are given in the B.A. Report for 1867. Since that date the coils have been repeatedly compared among themselves, and also with others of the original copies; and, with one exception, the apparent changes in their relative values, if any have occurred, are exceedingly small, and could be accounted for by the supposition that the temperature of the coil at the time of observation was uncertain to about ·1° C. Since, then, it is exceedingly improbable that all these coils of such different materials

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* "On the Specific Resistance of Mercury," 'Phil. Trans.,' 1883.

† 'Journal de Physique,' June, 1884.

‡ 'WIEDEMANN, Annalen,' vol. 25, 1885.

§ 'WIEDEMANN, Annalen,' vol. 25, 1885.

‖ Communicated to the British Association, 1887.

¶ 'Abhandl. der k. Bayer. Akad. der Wissenschaften,' II. Classe, vol. 16, Abth. iii.

** 'Comptes Rendus,' June 4, 1888.

should have changed by exactly the same amount during the last twenty-one years, it is inferred that no change has taken place in them, and the B.A. unit is defined as the mean of the values of the six coils at the temperatures at which they were said by HOCKIN (B.A. Report, 1867) to be correct. It was used in this sense by Lord RAYLEIGH in his electrical papers ('Phil. Trans.,' 1881, &c.), and this is the meaning attached to it in the various reports of the Electrical Standards Committee since the year 1882.

The method employed in making the observations differed but little from that given in Lord RAYLEIGH'S paper. The resistance at 0° Cent. of the column of mercury filling the tube is determined, in B.A. units; the length, L, of the column of mercury is measured; its mean cross section is found by measuring at a known temperature the length, l, of a column nearly filling the tube, and then finding the mass of mercury in the column. The mean cross section thus found needs correction for irregularities in the tube, and these are obtained by the ordinary process of calibration. The formula, as given in MAXWELL'S 'Electricity and Magnetism' (vol. 1, § 362), requires a small correction, for the fact that the length of the column used to determine the cross section does not quite fill the tube.

Let s be the cross section at a distance x from one end. Let λ be the length of a thread of mercury, which is passed along the tube, when its middle point is at a distance x from one end. Then, assuming the cross section to be constant over the length λ, we have $s = C/\lambda$, where C is the constant volume occupied by the thread; hence, if n be the number of points at which λ is measured and ρ the density of mercury in grammes per c.c.,

$$W = \rho \Sigma \left\{ s \frac{l}{n} \right\} = \rho C \Sigma \left(\frac{1}{\lambda} \right) \frac{l}{n}.$$

Again, let s_1 be the average cross section of the tube over the portion $(L - l)$ at the end which is not occupied by the mercury used to find the average cross section, and let $\bar{s}$ be the average cross section of the rest of the tube.

Then,

$$\bar{s} = \frac{C}{n} \Sigma \left(\frac{1}{\lambda} \right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and, if r be the resistance of a column of mercury 1 metre long, 1 square mm. in section, at 0° Cent., R the measured resistance of the tube,

$$R = \frac{r}{10^4} \left\{ \frac{1}{C} \Sigma (\lambda) \frac{l}{n} + \frac{L - l}{s_1} \right\} = \frac{r}{10^4} \left[\frac{1}{C} \Sigma (\lambda) \frac{L}{n} + \left\{ L - l \right\} \left\{ \frac{1}{s_1} - \frac{1}{nC} \Sigma (\lambda) \right\} \right]; \quad (2)$$

therefore,

$$WR = \frac{r\rho}{10^4} \left[\Sigma (\lambda) \Sigma \left(\frac{1}{\lambda} \right) \frac{Ll}{n^2} + l \left\{ L - l \right\} \left\{ \frac{C}{ns_1} \Sigma \left(\frac{1}{\lambda} \right) - \frac{1}{n^2} \Sigma (\lambda) \Sigma \left(\frac{1}{\lambda} \right) \right\} \right]; \quad (3)$$

so that, if we write μ for

$$\frac{1}{n^2}\Sigma(\lambda)\Sigma\left(\frac{1}{\lambda}\right),$$

we obtain

$$\mathrm{WR} = \frac{r\rho\mu\mathrm{L}l}{10^4}\left\{1 + \frac{\mathrm{L}-l}{\mathrm{L}}\left(\frac{s}{\mu s_1} - 1\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$r = 10^4 \times \frac{\mathrm{WR}}{\rho\mu\mathrm{L}l}\left\{1 - \frac{\mathrm{L}-l}{\mathrm{L}}\left(\frac{s}{\mu s_1} - 1\right)\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The term depending on $(\mathrm{L} - l)/\mathrm{L}$ is, of course, extremely small, but in some of the tubes employed it exercised a sensible effect on the result.

In addition to the corrections necessary to reduce the results to the standard temperature 0° C., the lengths L and l require corrections of importance.

The extremities of the tube opened into two large ebonite cups which were filled with mercury, and the observed resistance R includes that of the mercury in these cups which is situated just beyond the ends of the tube. Lord RAYLEIGH has shown that, on the assumption that the diameter of the mercury column in the cups is infinitely large compared with that of the tubes, the correction required would be equivalent to adding to the length of the tube ·82 of the diameter. The experiments of MASCART, NERVILLE, and BENOIT, 'Résumé des Expériences sur la Détermination de l'Ohm,' Paris, Gauthier-Villars, 1884, have justified this theoretical conclusion.

To make more certain on this point, we decided to repeat MASCART's experiments. A third cup was made, of the same size as the two terminals which are described later, differing from them only in having two openings, such as c, Plate 19, fig. 2. A tube, 95·8 cm. long, and about 1 B.A. unit in resistance, was taken. Its resistance when filled with mercury in the usual way was found. Its diameter, as found from its length and resistance, was 1·08 mm.; and therefore the theoretical correction for the two ends is equivalent to an addition to the length of ·886 mm., or to the resistance of ·00092 B.A. unit. The tube was then cut in two pieces, and an end was inserted in each of the two openings in the third mercury cup. This was filled with mercury, corked up, a thermometer passing through the cork, and replaced in the trough with the usual packing of ice. When the whole had cooled down its resistance was again measured and found to have increased by ·00089 B.A. unit. This increase is due to the resistance of the two cut ends, and the difference in the observed and the theoretical values is within the errors of the determination.

Another determination was made with a tube of about twice the cross section. In this case the theoretical correction was equivalent to an increase of resistance of ·00061 B.A. unit, while the observed was ·00059. It will be noticed that in both cases the observed value was less than the theoretical. This result was also found to be the case by LORENZ, who gives as the factor in the correction deduced from his experiments, the value $\cdot 82 - \cdot 35\, d_1/d_2$, d_1 being the diameter of the bore, and d_2 the outer diameter of the tube. In the case of the last tube mentioned above we had $d_1 = 1\cdot6$, $d_2 = 6$ mm., and LORENZ's term $\cdot35\ d_1/d_2$ has the value ·09, so that, according to him, for this tube, the correction should be $\cdot73\ d_1$. Our experiments would make this to be too small, though there is some evidence for a rather smaller coefficient than ·82. At the same time, it is hardly sufficient to justify any change, and we shall therefore add to the observed length of the tube a quantity δL equal to ·82 of the diameter of the tube.

The correction to the length l is not quite so simple. It arises from the fact that the ends of the column are not plane surfaces at right angles to its length, but portions of a curved surface which is spherical only if the tube, which was placed in a horizontal position, be so narrow that the effect of gravity may be neglected. The length l is the extreme length of the mercury column measured from end to end, and the volume found from it is therefore too great by the amount contained between the mercury and two vertical planes touching the mercury column at the extremity of each meniscus respectively.

This volume may be expressed in the form $\pi a^2 \delta l$, where a is the radius of the tube and δl a correction to be subtracted from the length. In cases in which the end of the mercury is spherical the calculation of δl is simple.

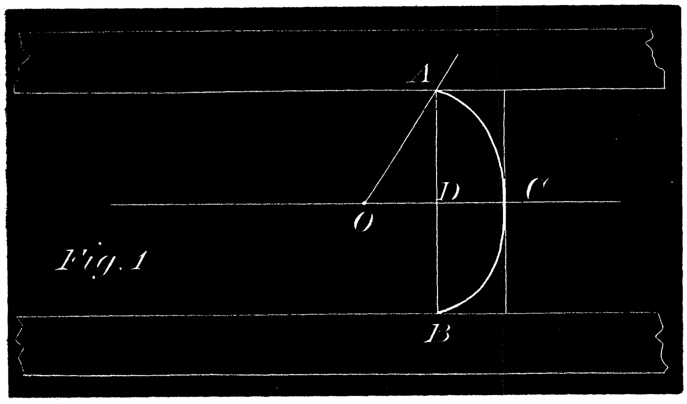

Fig. 1

Let ACB, fig. 1, represent a section of the mercury meniscus by a vertical plane through the axis of the tube; let CD be the axis of the tube, meeting AB at right angles in D; let the angles of contact at A and B, between the mercury and the glass of the tube, be θ; and let O be the centre and b the radius of the mercury bubble.

Then

$$\mathrm{OA} = b, \qquad \mathrm{DA} = a,$$

and

$$\text{Angle DAO} = \theta.$$

The length of the mercury column was determined by reading microscopes, as will be explained below, and in all cases the readings for both A and C were taken so that the distance CD was measured. This distance should, of course, be constant for any given tube, and experiment showed that it was nearly so. Let us put CD $= c$.

Then we have

$$\sin\theta = \frac{\text{OD}}{\text{OA}} = \frac{b-c}{b},$$

$$\cos\theta = \frac{\text{AD}}{\text{OA}} = \frac{a}{b}.$$

Hence,

$$\frac{c}{a} = \cot\tfrac{1}{2}\left(\frac{\pi}{2}+\theta\right).$$

Now, a is known with sufficient accuracy from the length and mass of the mercury column, so that the above equation gives us θ.

The following Table gives the values observed:—

TABLE II.

Tube	No. of observations.	c in cm.	a in cm.	θ
VI.	18	·026	·059	42°·30
VIII.	12	·029	·062	40·50
II.	8	·025	·055	40·20
IV.	2	·024	·057	44·20

The mean value of θ, allowing for the number of observations in each tube, is 41°·45. The angle of contact between mercury and glass is usually given as 42°, so that the agreement is very good.

Now, the volume of the spherical segment ACB is easily seen to be

$$\frac{\pi a^3}{3\cos^3\theta}(2 - 3\sin\theta + \sin^3\theta),$$

while that of the cylinder on AB as base and of height CD is

$$\frac{\pi a^3}{\cos\theta}(1-\cos\theta).$$

The correction, therefore, to be subtracted for each end of the tube from the whole volume $\pi a^2 l$ is the difference between these two, and this

$$= \frac{\pi a^3}{3\cos^3\theta}\{1 - 3\sin^2\theta + 2\sin^3\theta\};$$

therefore,

$$\delta l = \frac{2a}{3\cos^3\theta}\{1 - 3\sin^2\theta + 2\sin^3\theta\}.$$

If we pnt $\theta = 41^\circ{\cdot}45$, as found above, then

$$\delta l = {\cdot}42 \times a,$$

and this is the value used for the tubes VI., VIII., and II. in the Tables which follow.

In the case of the wider tubes, it was not at first so clear what the amount of this correction ought to be. If ACB, fig. 2, again represent the meniscus, it was evident that B was not vertically below A, and, further, that the extreme point C did not lie on the axis of the tube.

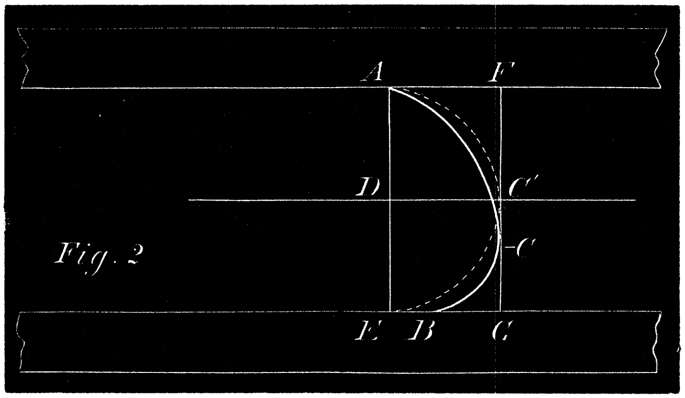

Fig. 2

Let AE be vertical through A, and FC′CG a vertical touching the end of the meniscus, and let DC′ be the axis of the tube.

Then, in the case of one tube of 1·9 mm. in diameter, we found that

$$\mathrm{AF} = {\cdot}46 \text{ mm.},$$
$$\mathrm{CC'} = {\cdot}11 \text{ mm.},$$
$$\mathrm{BG} = {\cdot}34 \text{ mm.},$$

approximately. Observations on other tubes gave somewhat similar values, though the difference between BG and AF was not always so great as in the above. The exact calculation of the volume between the plane through FG and the meniscus ACB is not possible. We have calculated the correction on the assumption that it is the same as that for a spherical surface through AC′E; the effect of gravity has been to draw this surface down into the position ACB, and it is assumed that the volume ACBE is approximately the same as AC′E.

Taking the values given above, we find on this assumption that

$$\delta l = {\cdot}46 \times a.$$

It seemed desirable, however, to verify this result by direct experiment, and this we did, following the method adopted by Lord Rayleigh. Ebonite plugs were turned which exactly fitted the ends of the tubes, and these plugs were inserted and pressed up against the ends of the mercury columns so as to flatten them, and the length of the column was measured with the reading microscopes. The ebonite plugs were then removed, and the full length of the mercury column measured. The difference between these two gives us δl directly; for one tube 1·9 mm. in diameter the mean of a number of determinations which were in fair agreement gave

$$\delta l = \cdot 45 \text{ mm.};$$

and for this tube we have, therefore,

$$\delta l = \cdot 47 \times a.$$

For a tube such as those used for the half units, for which the diameter was 1·57 mm., we found

$$\delta l = \cdot 35 \text{ mm.},$$

and this gives

$$\delta l = \cdot 45 \times a.$$

It is, therefore, clear that for these tubes we may, without serious error, use the value given by the above theory, viz.,

$$\delta l = \cdot 46 \times a,$$

and this has been done in the calculations.

Thus, the equation to determine r becomes

$$r = \frac{\text{WR} \times 10^4}{\rho\mu\,(\text{L} + \delta\text{L})\,(l - \delta l)} \left\{1 - \frac{\text{L} - l}{\text{L}} \left(\frac{\bar{s}}{\mu s_1} - 1\right)\right\}; \quad . \quad . \quad . \quad . \quad (6)$$

and, as will be seen when the values of the various quantities involved are introduced, this may be written

$$r = \frac{\text{WR} \times 10^4}{\rho\mu\text{L}l} \left\{1 - \frac{\delta\text{L}}{\text{L}} + \frac{\delta l}{l} - \frac{\text{L} - l}{\text{L}} \left(\frac{\bar{s}}{\mu s_1} - 1\right)\right\}. \quad . \quad . \quad . \quad . \quad (7)$$

In this expression temperature corrections are necessary to ρ, L, and l, while the weight W will require reducing to its value *in vacuo*.

Let

t'° = temperature at which the length L is measured;
b = coefficient of linear expansion of measuring rod = ·000017;
t = temperature at which the thread of length l is measured;
ρ* = density of mercury at 0° = 13·5957 grammes per c.c.;
γ* = coefficient of expansion of mercury = ·000182;
g = coefficient of cubical expansion of glass = ·000025.

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* These values are taken from the 'Travaux et Mémoires du Bureau International des Poids et Mesures,' vol. 2. See 'Nature' for April 3, 1884.

Hence,

$$\text{Volume of thread at } 0^\circ = \mathrm{W}/\rho.$$

$$\text{Volume of thread at } t^\circ = \frac{\mathrm{W}\,(1+\gamma t)}{\rho}\,.$$

$$\text{Mean section of tube at } t^\circ = \frac{\mathrm{W}\,(1+\gamma t)}{\rho l\,(1+bt)}\,.$$

$$\text{Mean section of tube at } 0^\circ = \frac{\mathrm{W}\,(1+\gamma t)}{\rho l\,(1+bt)\,(1+\tfrac{2}{3}gt)}\,.$$

$$\text{Length of tube at } 0^\circ = \frac{\mathrm{L}\,(1+bt')}{(1+\tfrac{1}{3}gt')}\,.$$

Hence, the value of $\mathrm{W}/\rho l\mathrm{L}$ corrected for temperature is

$$\frac{\mathrm{W}}{\rho l\mathrm{L}}\,\frac{(1+\gamma t)\,(1+\tfrac{1}{3}gt')}{(1+bt')\,(1+bt)\,(1+\tfrac{2}{3}gt)}\,;$$

and, if W_0 be the weight *in vacuo*, σ the density of air, and ρ' of the brass weights used,

$$\begin{aligned}\mathrm{W}_0 &= \mathrm{W}\left\{1-\sigma\,\frac{\rho-\rho'}{\rho\rho'}\right\}\\ &= \mathrm{W}\,\{1-{\cdot}000062\},\end{aligned}$$

taking dry air at 10°, the mean temperature of the weighings, and putting $\rho' = 8{\cdot}1$.

Hence, finally we get, introducing all the corrections,

$$r = \frac{10^4\,\mathrm{RW}}{\rho\mu\, l\mathrm{L}}\left\{1-\frac{\delta\mathrm{L}}{\mathrm{L}}+\frac{\delta l}{l}-{\cdot}000062-\frac{\mathrm{L}-l}{\mathrm{L}}\left(\frac{\bar{s}}{\mu s_1}-1\right)\right.$$
$$\left.+\left(\gamma-\tfrac{2}{3}g-b\right)t-\left(b-\tfrac{1}{3}g\right)t'\right\}; \quad . \quad . \quad . \quad . \quad (8)$$

and, if we put in numerical values for the one unit tubes, we have

$$r = \frac{10^4\,\mathrm{RW}}{\rho\mu\, l\mathrm{L}}\left\{1-{\cdot}82\,\frac{2a}{\mathrm{L}}+{\cdot}42\,\frac{a}{l}-{\cdot}000062-\frac{\mathrm{L}-l}{\mathrm{L}}\left(\frac{\bar{s}}{\mu s_1}-1\right)\right.$$
$$\left.+{\cdot}000149\,t-{\cdot}000009\,t'\right\}; \quad . \quad . \quad . \quad . \quad (9)$$

while, for the other tubes, the third term is $\cdot 46a/l$. We may write this

$$r = \frac{10^4\,\mathrm{RW}}{\rho\mu\, l\mathrm{L}}\,(1+\Delta), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where Δ is a small fraction, being the sum of all the correcting terms with their proper signs.

The methods used in finding L and l differed but little from those employed by Lord RAYLEIGH. The tubes were supplied by Messrs. POWELL and SON, and a number were roughly calibrated. The best of these were selected and were cut so as to have resistances of very approximately 1 B.A. unit, 1 legal ohm, $\frac{1}{2}$ B.A. unit, and $\frac{1}{3}$ B.A. unit respectively. The ends of the tubes were ground square by placing each in a groove in a piece of hard wood and allowing the end to pass through a hole which it accurately fitted in a brass plate fixed at right angles to the groove.

The tube was made to rotate slowly round its axis, and the end ground by emery on a copper plate.

To find the value of the length (L), two small rectangular pieces of brass were used. These were carefully squared, and had a fine × engraved on a small piece of white metal inserted in the centre of one of the faces of each block.

The tube was placed under two reading microscopes, which could be adjusted longitudinally by micrometer screws, graduated to ·00002 inch, and one of the rectangular pieces of brass was brought up to each end, and adjusted so that one edge touched the tube as nearly as could be along the horizontal diameter while the cross mark on the brass lay on the axis of the tube produced. The microscopes were then focussed on the crosses, and several readings taken as the tube was turned round on its axis; except in the case of one tube (No. III.), no appreciable difference was observed on turning the tubes round, and in the case of III. the difference did not amount to ·004 centimetre.

The tube and brass pieces were then removed and their places taken by the standard metre of the Cavendish Laboratory.

This is a bronze rod 1 metre long, divided into decimetres; the first decimetre is divided into millimetres. The length of the rod, as determined at the Standards Office of the Board of Trade by comparison with the standard metre, S.S., is 99·995 cm. ± ·001 at 0° C., while its coefficient of expansion is ·000017 per 1° C.

The first decimetre and the millimetre subdivisions have no appreciable error.

For the lengths greater than one metre a second bar, made by the Cambridge Scientific Instrument Company, was used. This was 30 cm. in length, and was divided into decimetres, which were compared with those of the standard.

The two bars were placed end to end in a long wooden box, which they fitted tightly, and the distance between the last division of the standard and the first division of the bar measured with the microscopes.

The temperatures were read by a thermometer laid on the scale or against the glass tube.

The length thus found was that of the glass tube, together with the sum of the distances between the cross marks on the brass blocks and their edges which were in contact with the tube. This distance was carefully determined by aid of the reading microscopes and the standard metre.

The values found for this correction differed at most by ·005 cm., and the observations on the length of the tubes, which were repeated in each case three or four times on separate occasions, agreed to about the same amount. Thus, the error in the length of the tubes is probably in no case greater than ·002 cm. in a length of about 100 cm.

The micrometer screws were only used to measure very small lengths, never so great as ·1 cm., and were tested and found without any error which could be appreciable.

The tubes were cleaned by passing through them in succession nitric acid, caustic potash, and distilled water, the last being repeated three times, alcohol, and finally ether, redistilled for the purpose. These were followed by air dried with chloride of calcium and passed through cotton wool. While the dry air was being passed through the tubes were heated with a spirit lamp, and then allowed to cool.

In several cases, following a suggestion of Professor ROWLAND'S, a plug of cotton wool was pushed through the tube with a wire, in order to loosen any small particles of dust which might adhere to the sides.

To calibrate the tubes, a short thread of mercury was inserted and moved into the various positions required ; its length λ being measured with the reading microscopes.

Table III. gives one set of readings for each tube. For the tubes VI., VIII., V., IX., the calibration observations were repeated, using threads of different lengths.

TABLE III.

VI.	VIII.	II.	V.	IX.	I.	III.
1·453	1·5654	1·5512	1·5464	1·3172	1·225	1·4334
1·439	1·5576	1·5860	1·5386	1·3220	1·219	1·4146
1·432	1·5452	1·6122	1·5386	1·3270	1·212	1·4130
1·432	1·5426	1·6400	1·5374	1·3310	1·208	1·4168
1·432	1·5452	1·6712	1·5428	1·3282	1·210	1·4234
1·430	1·5470	1·6742	1·5428	1·3234	1·214	1·4110
1·420	1·5470	1·6680	1·5560	1·3202	1·214	1·4010
1·416	1·5380	1·6588	1·5610	1·3170	1·212	1·3844
1·423	1·5360	1·6732	1·5648	1·3156	1·214	1·3670
1·422	1·5396	1·6688	1·5730	1·3216	1·218	1·3656
1·426	1·5322	1·6844	1·5882	1·3226	1·225	1·3690
1·429	1·5192	1·6908	1·5840	1·3234	1·224	1·3706
1·430	1·5188	1·6956	1·5758	1·3210	1·222	1·3706
1·443	1·5178	1·6802	1·5746	1·3270	1·221	1·3706
1·453	1·5292	1·6832	1·5784	1·3284	1·219	1·3720
1·453	1·5370	1·6782	1·5824	1·3356	1·214	1·3686
1·458	1·5446	1·6716	1·5786	1·3412	1·212	1·3576
1·464	1·5480	1·6558	1·5786	1·3430	1·211	1·3552
1·464	1·5576	1·6676	1·5704	1·3436	1·210	1·3552
1·464	1·5576	1·6788	1·5664	1·3426	1·209	1·3676
1·466	1·5532	1·6772	1·5674	1·3470	1·212	1·3660
1·467	1·5532	1·6720	1·5704	1·3500	1·206	1·3750
1·478	1·5460	1·6612	1·5720	1·3554	1·208	1·3906
1·484	1·5292		1·5756	1·3540	1·208	1·4006
1·489	1·5166		1·5756	1·3524	1·205	1·4122
1·498	1·5050		1·5692	1·3530	1·206	1·4094
1·507	1·4768		1·5528	1·3530		1·3976
1·512	1·4584		1·5420	1·3556		1·3882
1·504	1·4430			1·3504		1·3910
1·511	1·4360			1·3440		
1·518	1·4316			1·3356		
	1·4316			1·3240		
	1·4280			1·3240		

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The mean length of the threads and the corresponding values of μ are given in Table IV.

The agreement in the two values is sufficiently satisfactory.

TABLE IV.

No. of tube.	Mean length of mercury column, in inches.	μ.	Mean length of mercury column, in inches.	μ.	Value of μ used.
VI.	1·459	1·00044	1·228	1·00046	1·00045
VIII.	1·593	1·00079	1·436	1·00078	1·00079
II.	1·661	1·00045	..	..	1·00045
V.	1·564	1·00016	1·484	1·00015	1·00016
IX.	1·725	1·00009	1·335	1·00012	1·00010
I.	1·214	1·00002	..	.	1·00002
III.	1·386	1·00032	..	..	1·00032

To find the average cross section of the tube a thread of mercury almost filling the tube was used. In nearly all cases this was the same thread as was used in finding the resistance. The extreme length (l) of the thread between its curved ends was found with the reading microscopes and the standard scale, while the curvature of the ends was found by reading the distance between the end of the meniscus and the point in which the mercury touched the glass. For a given tube this distance did not vary very greatly in most of the observations, and, with one exception, VI. (2), the mean value has been taken in calculating the correction δl. The method of determining this correction has already been given. The temperature was observed by means of a thermometer laid alongside the tube, which was generally left in position for some hours before observations commenced. The microscopes and thermometer were then read at intervals of about 15 minutes, and, when two or more consecutive values for both were found to be the same, it was supposed that the temperature of the mercury was that given by the thermometer. This was verified in several instances by taking the temperature of the mercury after it had been allowed to run from the tube into the small crucible in which it was weighed.

As has already been stated, the mercury thread did not entirely fill the tube, and, in consequence, a small correction was needed. The amount of this correction is $\frac{L-l}{L}\left(\frac{\bar{s}}{\mu s_1}-1\right)$, $\bar{s}$ being the mean cross section, and s_1 the mean of the cross sections at the ends.

To determine the ratio $\bar{s}/s_1$ a point was found on the tube from the calibration experiments at which the actual cross section was equal to the mean value. A short thread of mercury, some 3 to 4 mm. in length, was introduced, and its length measured when its middle point was at the point of mean cross section: let this length be $\bar{l}$. The thread was then moved to one end of the tube and its length measured in various positions close up to the end. The same process was repeated at the other end: let the mean of the lengths thus found be l_1. Then, if we suppose that the curvature of the meniscus did not alter (and experiment showed that this assumption is nearly true), we have very approximately the equality

$$\frac{\bar{s}}{s_1} = \frac{l_1}{\bar{l}}.$$

Thus, in tube VI., which is slightly smaller at both ends than it is in its central part, the value of $\bar{l}$ is 3·63 mm., while the length of the thread was the same at each end, and was equal to 3·78 mm. For this tube the mean value of $(L - l)/L$ was ·00367, and the average value of the correction ·000150. It is larger in the case of this tube than with any other used, for in most the effect of one end was opposite to that of the other.

Table V. gives the values of the lengths of the threads at the two ends respectively, the mean of these two or l_1, and of $\bar{l}$ for the various tubes.

TABLE V.

Tube.	Length at one end.	Length at second end.	l_1.	$\bar{l}$.
VI.	3·78	3·78	3·78	3·63
VIII.	4·72	4·33	4·53	4·58
II.	3·63	3·87	3·75	3·87
V.	3·90	3·89	3·90	3·94
IX.	3·48	3·53	3·51	3·45
I.	Tube	uniform		
III.	5·68	5·56	5·62	5·50

The mercury was weighed, using a balance by OERTLING and the weights employed by Lord RAYLEIGH; these were compared with each other and with a set of weights which Mr. SHAW had previously compared with the standard 500-gramme weight of the Laboratory. A small correction of about ·1 milligramme on 10 grammes was found, and has been introduced, but it is too small to affect our results.

The mercury was weighed in both pans of the balance, and the weighings repeated on two or more different days.

In the electrical measurements the tubes were compared directly by CAREY FOSTER'S method with the B.A. standards, using the bridge designed by Dr. FLEMING which was employed by Lord RAYLEIGH. The tubes had been so adjusted that the difference between them amounted only to a few centimetres, at most 70 divisions of the bridge wire. We were thus independent of variations in the resistance of the wire due to temperature changes, and the value of the bridge wire division was taken as ·0000498 B.A. unit.

The ends of the mercury tube were connected, in a manner to be described shortly, to the bridge by copper rods, and rods of the same material and almost the same resistance were used to connect the standard coils. It was hoped in this manner to compensate the effect produced on the resistance of these rods by changes of temperature in the room. Since the difference between the two sets of rods was only equivalent to one bridge wire division, this was completely secured.

The coils used were the following :—For the tubes VI. and VIII. the standard F was employed; for V. and IX. the standards F and G in multiple arc; and for I. and III. F, G, and Flat in multiple arc.

In the case of the legal ohm, tube II., a coil of 100 B.A. units, ELLIOTT No. 68, was placed in multiple arc with the tube, and the difference between the combination and F was found in the usual way. The temperature of the water baths in which the coils were placed was taken with a thermometer which had been compared at Kew, and the necessary corrections applied. The temperature of the baths and of the room in which the experiments were made never differed greatly from 10°. The following Table VI. gives the values of the coils at 10°, with their temperature coefficients. The value of F and G are taken from Dr. FLEMING's chart. The difference between the two coils at the time of the observations was determined and found to agree exactly with that given by the chart. In the case of Flat, which is not one of the six coils mentioned on p. 352, repeated observation during the last two years has shown that it is now slightly lower relatively to the others than when examined by Dr. FLEMING; the change is not greater than ·0001 B.A. unit, and is probably due to a slight imperfection in the insulation. We have taken the value relative to F and G given by our own observations; as Flat is only used in multiple arc with F and G for the tubes I. and III., any uncertainty in its value is divided by nine in the result, and the error introduced is too small to trouble us.

The coil of 100 B.A. units is one of the standards of the Association, and, like the other coils, is of platinum-silver wire.

TABLE VI.

Coil.	Value at 10°.	Coefficient.
F	·99807	·000272
G	·99778	·000263
Flat	·99857	·000277
No. 68	99·847	·0270

For the one unit tubes the terminals of F dipped into mercury cups on ebonite, which were connected to the bridge by the copper rods above mentioned. When the coil G was used its terminals dipped into the same cups, and for the one-third unit tubes I. and III., the coil Flat dipped into two other cups connected with the first two by thick pieces of copper. The resistance of these connexions was determined by finding the difference between Flat and F directly, and then when Flat was connected to the bridge by these copper pieces. In this way we found the resistance of the connexions to be ·00136 B.A. unit. The temperature was about the same as that at which the connexions were used. In the values of Flat given in Table VIII. the resistance of these connexions has been included.

In all cases the temperature of the room was almost exactly the same as that of the water baths.

One of the ebonite cups into which the ends of the tubes opened is shown in figs. 1 and 2, Plate 19, which is drawn to scale full size. In their design two points mainly were attended to. The first was, that it should be possible to reduce the mercury in them very nearly to 0° C.; the second, that there should be no contact between copper and mercury, for BENOIT has shown that the conductivity of mercury is in a very short time appreciably increased by contact with copper.

The glass tube passes through an india-rubber cork, which fits into the terminal at c, c_1; the tube was usually adjusted so that its end was flush with the inner surface of the terminal.

Mercury was then poured into the cup and allowed to run slowly through the tube into the second terminal until each was about two-thirds full. The top shown in fig. 1 was then placed over the terminal and secured by four small screw-bolts passing through the flange *a*, *a*. When these were screwed down the terminals were completely water-tight, and could be left covered with melting ice or water for days without leakage.

The top consists of a flat plate of ebonite, with four holes to receive the bolts. Through this plate two ebonite tubes, *dd*, *ee* (figs. 1 and 3) pass. A hollow platinum cup, *f*, about 3·5 cm. long by rather more than 1 cm. in diameter, is secured firmly into the tube *d*, *d*, and a thick piece of copper rod, *g*, fits the interior of the cup tightly, any interstices between the two being filled with mercury; the surface of the copper rod was well amalgamated. This copper rod, *g*, is brazed to the copper rods, *g′*, which form the connexion with the bridge. Pieces of india-rubber tubing, *hh*, *kk*, about 10 cm. long, are fastened over the upper ends of the tubes, *dd*, *ee*; the connexion to the bridge passes through a cork, which closes the upper end of the tube, *hh*, and the junction is made water-tight with marine glue. A thermometer, *t*, graduated to fifths of a degree Centigrade, passes through *ee* and the india-rubber tube, *kk*, which fits it closely, and gives the temperature of the mercury in the terminal.

In taking the observations the ice was packed closely round the terminals up to the tops of the tubes, *h* and *k*, so that the copper rods were surrounded by ice for 12 or 14 cm. above the level of the mercury. Contact between the copper and the mercury in the terminals was thus established through the platinum cup, *f*; the surface of this is about 12 sq. cm. This surface was amalgamated in the following manner:—The cups were platinised by electrolysis from a solution of platinic chloride in nitric acid. On immersing the cups in mercury, after heating to drive off traces of the acid, amalgamation readily took place.

After this process thoroughly good contact between the platinum and the mercury was secured. To test this, the two platinum cups were placed in the same vessel of mercury, and the ends of the copper rods connected with the bridge. In this position the resistance of the connexion was measured, and gave the same value before and after the experiments, while no appreciable change could be noticed on taking one of the platinum cups out of the mercury and again replacing it. When only about one-third of each platinum cup was in the mercury, the resistance was increased by about ·00004 B.A. unit. In use care was taken to place sufficient mercury in the terminals to cover the cups entirely. The following additional experiment shows the goodness of the contact: The platinum cups were placed in the mercury, and the resistance measured as described; then the copper rods, g, were removed from the interior of the cups and placed in the same vessel of mercury, the other ends of the copper rods being in connexion with the bridge, and the resistance was again measured. No difference between these two measurements could be detected.

Thus, the contact through the platinum was practically as good as if the copper rods, g, had dipped directly into the mercury in the terminals.

Tables VII. and VIII. give details as to the mercury employed. The general method of treatment was as follows:—Mercury from the ordinary stock in the Laboratory was treated with nitric acid and potash, and then distilled *in vacuo* in the Laboratory still. This mercury after being once distilled was again mixed with nitric acid, being allowed to stand overnight in contact with it. After this it was heated with caustic potash, and then well washed and dried by being strongly heated, and finally it was passed through a second still, newly set up for the purpose, in which only mercury which had been previously distilled and treated as above was ever placed.

TABLE VII.

Date of electrical observation.	No. of tube.	No. of filling.	W.	l.	L.	a.	μ.	t.	t'.	$(\gamma-\frac{2}{3}g-b)\,t$ $=\cdot000149t$	$(\frac{1}{3}g+b)t'$ $=\cdot000009t'$	$\frac{\delta L}{L}$	$\frac{\delta l}{l}$	$\frac{L-l}{L}\left(\frac{\bar{s}}{s_1\mu}-1\right)$	Value of Δ.
Dec. 23rd	VI.	1	16·5333	112·772	113·134	·0586	1·00045	12·0°	9·3°	·001788	·000083	·000848	·000217	·000130	·000884
„ 28th	„	2	16·5913	113·058	„	„	„	5·7	„	·000849	„	„	·000162	·000027	—·000008
„ 29th	„	3	16·5322	112·763	„	„	„	12·1	„	·001803	„	„	·000217	·000183	·000846
Jan. 2nd	„	5	16·5456	112·807	„	„	„	9·1	„	·001356	„	„	„	·000118	·000464
„ 3rd	„	6	16·5244	112·641	„	„	„	8·4	„	·001252	„	„	„	·000178	·000300
„ 9th	„	8	16·5306	112·716	„	„	„	10·2	„	·001520	„	„	„	·000150	·000596
Dec. 23rd	VIII.	1	20·9836	127·036	127·438	·0622	1·00079	10·3	10·7	·001535	·000096	·000798	·000205	—·000040	·000826
„ 28th	„	2	20·8629	126·239	„	„	„	6·3	„	·000939	„	„	„	—·000119	·000309
„ 29th	„	3	20·9536	126·801	„	„	„	8·4	„	·001252	„	„	„	—·000063	·000566
Jan. 2nd	„	5	20·9670	127·043	„	„	„	17·0	„	·002533	„	„	„	—·000039	·001823
„ 9th	„	7	20·9764	126·931	„	„	„	8·1	„	·001207	„	„	„	—·000050	·000508
Jan. 2nd	II.	1	13·2567	101·546	101·904	·0553	1·00045	7·9	11·0	·001177	·000099	·000890	·000229	—·000110	·000467
„ 3rd	„	2	13·2149	101·237	„	„	„	7·9	„	·001177	„	„	„	—·000206	·000563
„ 5th	„	3	13·2543	101·540	„	„	„	9·7	„	·001445	„	„	„	—·000112	·000737
„ 10th	„	4	13·2707	101·630	„	„	„	6·9	„	·001028	„	„	„	—·000085	·000293

Date of electrical observation.	No. of tube.	No. of filling.	Standard used.	Temperature of standard.	Value of standard.	Difference between standard and R.	R.	Temperature of end pieces.	Mercury used.	r.	Mean value of r.
Dec. 23rd	VI.	1	F	11·9°	·998586	·001424	1·000010	2·2°	Laboratory Hg, twice distilled	·953584	·95357
„ 28th	„	2	„	8·7	·997716	·002233	·999949	1·2	„ „	·953640	
„ 29th	„	3	„	8·8	·997743	·002206	·999949	1·2	„ „	·953527	
Jan. 2nd	„	5	„	9·9	·998043	·001872	·999915	1·2	Same treated with nitric acid	·953506	
„ 3rd	„	6	„	10·3	·998152	·001830	·999982	1·1	„ „	·953595	
„ 9th	„	8	„	10·5	·998207	·001748	·999955	1·1	Hg from Chem. Lab., redistilled . . .	·953574	
Dec. 23rd	VIII.	1	F	12·0	·998614	·001519	1·000133	2·5	Laboratory Hg, twice distilled	·953506	·95354
„ 28th	„	2	„	8·65	·997703	·002400	1·000103	1·2	„ „	·953485	
„ 29th	„	3	„	9·1	·997825	·002171	·999996	1·2	„ „	·953530	
Jan. 2nd	„	5	„	9·5	·997934	·002106	1·000040	1·4	Hg as supplied by manufacturers . . .	·953560	
„ 9th	„	7	„	11·7	·998532	·001504	1·000036	1·7	Hg from Chem. Lab., redistilled . . .	·953605	
Jan. 2nd	II.	1	F	9·5	·997934	·013992	1·011926	1·1	Hg treated with nitric acid and redistilled	·953528	·95349
„ 3rd	„	2	„	10·4	·998179	·013626	1·011805	1·4	„ „	·953400	
„ 5th	„	3	„	10·4	·998179	·013631	1·011810	1·3	„ „	·953560	
„ 10th	„	4	„	11·1	·998367	·013434	1·011803	1·5	„ „	·953464	

Tables VII. and VIII., which contain the results of the observations, do not require much explanation. The one unit tubes are given in Table VII. In tube VI. filling 2, the curvature of the ends of the mercury column was much less than in any other case, and the correction $\delta l/l$ has, therefore, been calculated specially for this tube The result is clearly too high, but there is no reason in the details of the measurements for omitting it. It may be noticed that the results of the 4th and 7th fillings are not given in the Table. These will be referred to again shortly; they were fillings of a special character, and it seemed best to treat them separately.

The extreme difference between any two fillings is ·000135, and the difference between the mean result and the extreme is about half of this. In some of the columns the figures have been given to six places; the only object in this is to secure accuracy in the fifth figure in the final result.

In the Table for tube VIII., the fourth and sixth fillings are omitted for a similar reason to that given above. The extreme difference between any two fillings is about the same as in the case of VI. The mercury used in VIII. 5 was taken directly from the iron bottle in which it was supplied by Messrs. TUBBS and WILKINS, and was not distilled in the Laboratory. In the first filling of VIII., and the first and third fillings of VI., the filling from which the cross section is determined was different from that used to determine the resistance. This was due to the fact that at first the necessity of having the tube almost completely full when finding l was not fully appreciated, and too much mercury was allowed to escape from the ends in removing the terminals. In the other cases in this table the two fillings were the same. The mean value for tube II. is reduced by the result of the 2nd filling, which is clearly too low, but the observations themselves do not give us any reason for rejecting it. All the fillings for this tube have been included.

The mean value of r found from these three tubes is ·95354 B.A. unit, and the greatest difference between any two observations is ·000241 B.A. unit. The average error independent of sign is ·00005.

If, however, we rejected the results of VI. 2 and II. 2, the mean would hardly be affected, but the mean error would be reduced to ·000037.

We may, therefore, fairly put as the result of the observations on the one unit tubes $r = \cdot 95354 \pm \cdot 00004$. This is, it will be observed, the value given by tube VIII. The value for VI. is raised unduly by the observation VI. 2; that for II. is unduly lowered by the result of II. 2.

TABLE VIII.

Date of electrical observation.	No. of tube.	No. of filling.	W.	l.	L.	a.	μ.	t.	t'.	$(\gamma-\frac{2}{3}g-b)t$ $=\cdot000149t$.	$(\frac{1}{3}g+b)t'$ $=\cdot000009t'$	$\frac{d\mathrm{L}}{\mathrm{L}}$.	$\frac{dl}{l}$.	$\frac{\mathrm{L}-l}{\mathrm{L}}\left(\frac{\bar{s}}{s_1\mu}-1\right)$.	Value of Δ.	Standard used.	Temp. of F.
Jan. 4th	V.	1.	32·9847	112·497	112·950	·0829	1·00016	8°·7	12°·3	·001296	·000111	·001204	·000339	− ·000041	·000299	F	10°·2
,, 5th	,,	2.	32·9556	112·388	,,	,,	,,	8·8	,,	·001311	,,	,,	,,	− ·000051	·000324	,,	10·7
,, 4th	IX.	1.	31·7186	110·439	110·913	·0820	1·00010	10·7	10·6	·001594	·000095	·001212	·000341	·000064	·000500	F	10·3
,, 5th	,,	2.	31·7086	110·369	,,	,,	,,	9·3	,,	·001386	,,	,,	,,	·000073	·000285	,,	10·8
,, 6th	I.	1.	32·6612	91·390	91·723	·0915	1·00002	12·2	12·3	·001818	·000111	·001636	·000461	. .	·000470	F	10·7
,, 10th	,,	2.	32·6675	91·338	,,	,,	,,	6·5	,,	·000968	,,	,,	,,	. .	−·000380	,,	10·8
,, 6th	III.	1.	40·5245	101·339	101·765	·0968	1·00032	12·0	12·7	·001788	·000114	·001559	·000439	·000090	·000402	F	10·9
,, 9th	,,	2.	40·5005	101·374	,,	,,	,,	17·3	,,	·002578	,,	,,	,,	·000083	·001199	,,	11·5

Date of electrical observation.	No. of tube.	No. of filling.	Value of F.	Standard used.	Temp. of G.	Value of G.	Standard used.	Temp. of Flat.	Value of Flat with connexions.	Value of coils in multiple arc.	Difference between R. and coils in multiple arc.	R.	Temp. of end pieces.	Mercury used.	r.	Mean value of r.
Jan. 4th	V.	1.	·998124	G	10°·2	·997833	. .	. .	. .	·498989	·000244	·499233	1°·3	Hg redistilled after treatment with nitric acid	·95334	·95338
,, 5th	,,	2.	·998260	,,	10·8	·997987	. .	. .	. .	·499061	·000154	·499215	1·4		·95341	
,, 4th	IX.	1.	·998152	G	10·4	·997786	. .	. .	. .	·499034	·001354	·500388	1·3		·95344	·95344
,, 5th	,,	2.	·998287	,,	11·0	·998040	. .	. .	. .	·499080	·001250	·500333	1·3		·95343	
,, 6th	I.	1.	·998260	G	10·8	·997987	Flat	10·6	1·000096	·332927	−·000388	·332539	1·1		·95343	·95344
,, 10th	,,	2.	·998287	,,	10·9	·998015	,,	10·7	1·000124	·332936	−·000361	·332574	1·3		·95345	
,, 6th	III.	1.	·998314	G	11·0	·998042	Flat	10·7	1·000124	·332939	−·003088	·329851	1·3		·95344	·95342
,, 9th	,,	2.	·998478	,,	11·5	·998174	,,	11·4	1·000318	·332996	−·003118	·329878	1·4		·95339	

Table VIII. gives the results of the observations on the one-half and one-third unit tubes. One other filling of I., which is not given, was taken, but it was clear that some accidental error had been made in the measurement of l or of W, for the result differed from the others by over one in a thousand. This is the only filling in the whole series which has been entirely rejected. The result of V. 1 is clearly too low.

If we give the result of all the fillings in Table VIII. equal weight, we obtain as the mean value $r = \cdot 95342$ B.A. unit, and the average error is ·000029.

The difference between this and the result derived from the one unit tubes is, at first sight, considerable, but it must be remembered that for the tubes I. and III. an error in the measurement of resistance of ·00003 or one thirty-thousandth of a B.A. unit gives rise to an error of one ten-thousandth in the result. This, of course, is an extremely small quantity. Moreover, the small uncertainty which attaches to the corrections δL and δl would produce a larger effect in these large tubes, and our observations tend to show that the coefficient ·82 in the value for δL is possibly rather too great.

It will be noticed, however, that the values of R, the actual observed resistance of a tube, differ among themselves by extremely small quantities.

In taking a final mean, it was clearly unfair to weight the observations equally, and we came to the conclusion that the probable accuracy was roughly inversely proportional to the area of the cross section. We have, therefore, attached the weights 3, 2, and 1 to the results from the one unit, half unit, and third unit tubes respectively, and arrive at the final result that the resistance of a column of mercury 1 metre long, 1 square millimetre in section, at 0° is

·95352 B.A. unit.

If we give equal weights to all the observations, the result will be ·95351, so that the effect of the weighting is hardly appreciable.

It remains to consider the four special fillings of tubes VI. and VIII., which have been omitted. In two of these, VI. 4 and VIII. 4, the attempt was made to fill the tube quite full, when measuring l; a very small bubble of mercury was left protruding from each end of the tube when it was placed under the reading microscopes, and then flat pieces of brass were brought up simultaneously against the ends, there being a layer of thin paper between the brass and the mercury. It was hoped in this way to squeeze out the superfluous mercury and leave a column with flat ends exactly filling the tube. It seems probable in the case of VI. 4 that this was successfully accomplished, for the value of r found from the experiment is ·95354 B.A. unit.

With VIII. 4, however, it was clear, on looking through the microscopes at the mercury column, that the ends in contact with the paper were slightly curved, and this was still more obvious when the brass and paper were removed. The resulting value of r is accordingly too low, being ·95342 B.A. unit.

In the case of the two fillings, VI. 7 and VIII. 6, mercury was used which had been passed once through the still at the University Chemical Laboratory, and then treated with nitric acid. It was clear, from the appearance of the mercury, that it was impure; but it was thought of interest to determine a value for mercury in the purifying of which no special trouble had been taken. The impurity shows itself at once in the results, for, while the mean value of R for VI. is ·99996, for this filling R = ·99989; while for VIII. the resistance of the filling is ·99990 B.A. unit, against a mean of 1·00006. The corresponding values for r are ·95348 and ·95329.

This mercury was then treated with nitric acid, &c., redistilled in our own still, and used again in the fillings VI. 8 and VIII. 7; the values of R were ·99996 and 1·00004, and of r ·95360 and ·95361 respectively.

Thus, the impurity has been clearly removed by the distillation and acid treatment.

In some cases a tube was filled on one afternoon, and its resistance determined. The whole was then allowed to stand over night, being re-packed in ice in the morning, and the resistance again measured, but no appreciable change was noted. Thus, for VI., on January 9, the value ·99994 was found, while the same filling, re-packed on January 10, after the mercury had stood for 16 or 18 hours in contact with the platinum of the terminals, gave R = ·99996; the difference is within the temperature errors of the coils.

Some experiments were made on the effect of known impurities on the mercury in altering its resistance. In one case, about one two-thousandth part of zinc filings was added to the mercury. On mixing, the surface of the mercury was made foul; the mercury was then passed through a filter paper and used in VI., but the effect on the resistance was not appreciable. It is probable, of course, that the filtering had removed a large portion of the zinc, but the experiment gives some idea of the amount of impurity which the resistance measures will detect.

In another filling of VI., a mixture of mercury with a small percentage of tin was used. The resistance was much too small to be measured on the bridge—the bridge wire has a resistance of ·05 B.A. unit approximately. This mercury was treated in the usual way, distilled in the Laboratory still, and then in our own, and on being again used gave as the value for R ·99991 B.A. unit. Thus our treatment was sufficient to remove the tin from the mercury.

Some observations were also made on the change of resistance of mercury with temperature.

Thus, on January 5, tube II. was placed in a trough in water at about the temperature of the room, and its resistance measured. The tube was then packed in ice and measured. Similar observations were made with tube III., and the results are given in the Table IX. Other observations confirmed the results there given.

TABLE IX.

Date.	Tube.	Temperature.	Value.	Value at 0°.	Coefficient.
January 3 . . .	II.	9·5	1·02028	1·01186	·000878
„ 9 . . .	III.	10·5	·33297	·32993	·000875

We conclude finally from the experiments that the value of

r is **·95352** B.A. unit.

If we take as the value of the B.A. unit the mean of those found at the Cavendish Laboratory, we have

1 B.A. unit = ·98667 Ohm,

we find that

r = **·94081** Ohm,

or 1 ohm is equal to resistance at 0° C. of a column of mercury 1 square millimetre in area and

106·29 centimetres in length.

These values agree closely with those communicated at Manchester to the British Association by Professor ROWLAND, viz.:

r = ·95349 B.A. unit.

1 ohm = 106·32 centimetres of mercury at 0° C.

The value of r in B.A. units does not differ greatly from that found at Wurzburg by STRECKER and by KOHLRAUSCH;* the difference, however, is greater than can be accounted for by error of experiment, but is, I think, capable of easy explanation.

STRECKER's comparison of his mercury tubes with one of the B.A. units sent from Cambridge was made at a temperature of from 9° to 10°·5 ('WIEDEMANN, Annalen,' vol. 25, p. 482). The resistance of the mercury at this temperature was reduced to 0° by means of his own formula (*loc. cit.*, p. 474), which gives a mean coefficient up to 10° of ·000909. Now, this is a larger value than is given by any other observer, as is shown in the following Table :—

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* See the Table on page 352.

TABLE X.

Observer.	Average coefficient up to 10°.
STRECKER *	·000909
SIEMENS †	·000865
LORENZ ‡	·000901
LENZ §	·000884
BENOIT ‖	·000877
RAYLEIGH ¶	·000861
GLAZEBROOK **	·000861
GLAZEBROOK and FITZPATRICK ††	·000876
Mean	·000879

Thus, STRECKER's value is higher than the mean by ·00003, and if we were to reduce his observations from 10° to 0°, using the mean coefficient given above, we should obtain the value for r, ·95362.

This value may possibly be a little too high, but at any rate the reasoning is sufficient to show that the difference may easily depend on a small error in the temperature coefficient.

The same reasoning will apply to KOHLRAUSCH's results, for his comparisons between the mercury tubes and the wire standards were usually made at temperatures differing from 0°, and were reduced to 0° by the use of STRECKER's formula.

The value of the ohm in centimetres of mercury at 0°, as given by KOHLRAUSCH in a letter to R. T. G., February 16, 1888, is 106·32, agreeing exactly with ROWLAND.

MASCART, NERVILLE, and BENOIT found a value for r which is as much above our value as KOHLRAUSCH is below it. At the same time, their value of the B.A. unit in ohms is less than ours, leading to the result that the value of the ohm in centimetres of mercury is 106·33; or, again, the same value as ROWLAND's. This might seem to show that there was some small change in the B.A. unit used by MASCART between the time it was compared at Cambridge and the date of their observations.

[Quite recently, June 4, 1888, M. WUILLEUMIER communicated to the Academy of Sciences at Paris the results of some experiments by LIPPMANN's method, which give the value 106·27.]

Thus we may conclude that the experiments of MASCART, STRECKER, ROWLAND, KOHLRAUSCH, WUILLEUMIER, and ourselves are in fairly close agreement, and that

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[*] 'WIEDEMANN, Annalen,' vol. 25, p. 475.

[†] 'Electrotechn. Zeitschr.,' vol. 3, 1882, p. 408.

[‡] 'WIEDEMANN, Annalen,' vol. 25, p. 11.

[§] 'Études Électrométrologiques,' vol. 2, 1884.

[‖] "Résumé d'expériences sur la détermination de l'ohm," 'Journal de Physique,' 1884, p. 230.

[¶] 'Phil. Trans.,' 1883, p. 185.

[**] 'Phil. Mag.,' October, 1885, p. 352.

[††] *Supra*, p. 372.

the value of the ohm expressed in centimetres of mercury at 0° does not differ from 106·31 by more than ·02 of a centimetre, or two in ten thousand.

From this result the values found by LORENZ and Lord RAYLEIGH differ appreciably. With regard to LORENZ's value, we may notice that the comparison between his tubes and the B.A. unit was very far from being direct.

The tubes were compared with a Siemens' unit issued by SIEMENS and HALSKE, and this with a copy of the B.A. units sent to LORENZ by Lord RAYLEIGH. The temperature coefficients of the two coils and of the mercury required to be known, and corrections introduced. The final value found by LORENZ for the ohm in centimetres of mercury is 105·93.

In this determination, tubes were used 1 metre long, and 1, 2, and 3 centimetres in diameter, and some part of the large difference may possibly be due to the fact that the lines of flow of the current near the ends of the tubes can hardly have been cylindrical.

No such explanation, however, can be offered of the difference between Lord RAYLEIGH'S result and our own. His comparisons were direct, and the results of the observations on the various tubes employed are extremely concordant; the tubes actually used by him have since been broken, but the end pieces are still at the Laboratory. We thought it was worth while to fit up one of our tubes, No. VI., with his end pieces, and find its resistance. In this way, we were able to eliminate any error which might have occurred in the resistance of connexions, as new connexions were made for the purpose and had their resistance specially determined.

The experiment was made on May 19th, and the value found, when the temperature in the terminals was about 3°, was

1·00000 B.A. unit.

The mean value for VI. previously found, the temperature in the end pieces being 1°·4 C., was

·99996 B.A. unit.

Thus, this experiment fully confirms the value we had already used, and shows that no error can have been introduced by the connexions. Lord RAYLEIGH has himself pointed out that the fact that the temperature of the mercury in his terminal cups was from 5° to 6° C. would lead to an over-estimate of the value of r, and he concludes that this over-estimate may in his case have been as much as ·00008.

In the course of our observations we had several times determined the resistance of a tube as the mercury in the end pieces cooled down from 9° or 10°, the temperature of the room, to the temperature at which it was finally steady, which was on the average about 1°·4 C.

Table XI. gives the results of these determinations for VI. Each horizontal line refers to the same filling. Taking the resistance when the mercury in the end pieces was at 2° as 1 B.A. unit, the Table gives the temperatures at which the resistances were measured, and the increase of the resistance of the tube up to the temperature in question.

TABLE XI.

Temperature . . . . Increase of resistance above that at 2°	2°·8 ·00004	4°·5 ·00010			8°·8 ·00035	10°·6 ·00043
Temperature . . . . Increase of resistance above that at 2°	2°·9 ·00003	4°·6 ·00010			9° ·00035	
Temperature . . . . Increase of resistance above that at 2°				6°·6 ·00020		
Temperature . . . . Increase of resistance above that at 2°				6°·2 ·00025		
Temperature . . . . Increase of resistance above that at 2°		4° ·00010	5°·5 ·00020			

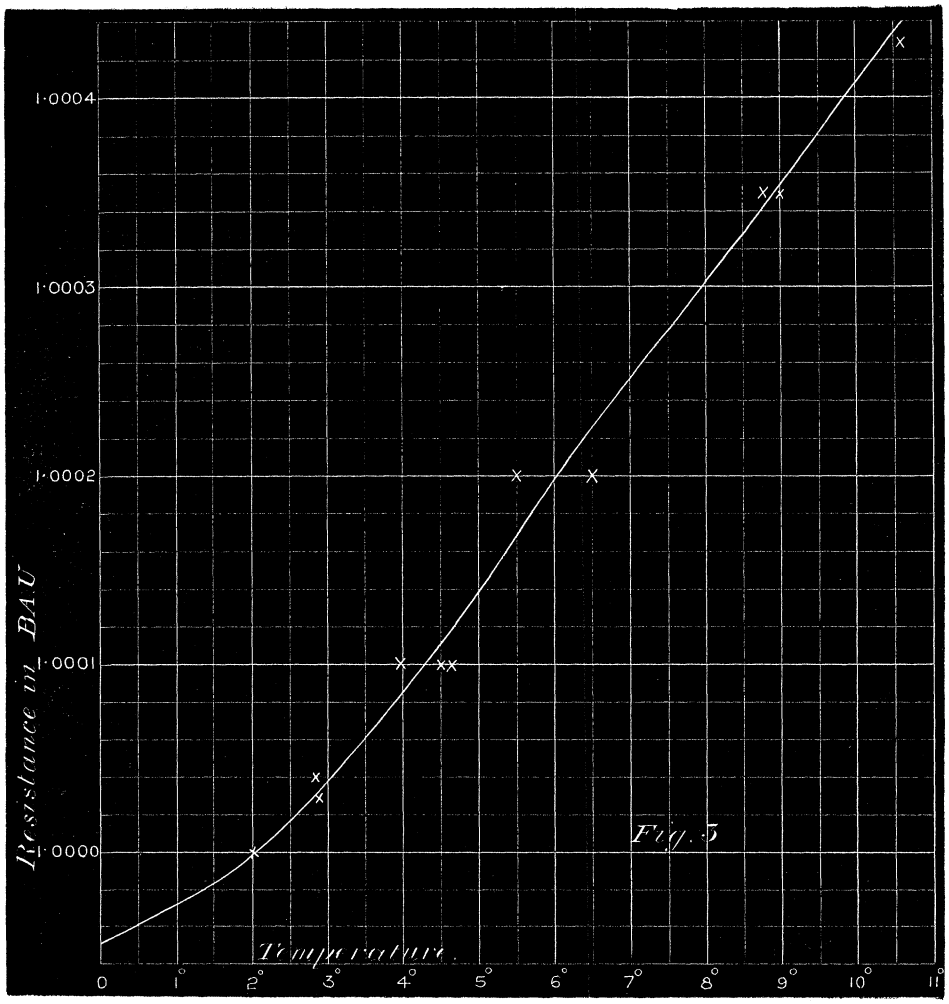

Fig. 3

The results of the Table are represented graphically in fig. 3, in which the abscissæ represent temperature and the ordinates resistance, the temperature being that indicated by the thermometers in the cups. The tube was of course packed in the ice during these observations. It would appear from the curve that our own observations may be slightly too high, possibly as much as ·00004, through the temperature in the cups being on the average 1°·4 C. instead of 0°, while at 6° an error of about ·00024 might be introduced. This error is equivalent to that caused by the whole tube being at 0°·3 instead of at 0°, or by about 5 per cent. of the tube being at the temperature of the mercury in the terminals. In tube VI. some 6 or 7 per cent. of the tube was within the corks used to close the terminals. It may be noticed that the observations on tube VI. given in the last line of Table XI. were made with Lord RAYLEIGH's terminals. We thus infer that, while the fact that in Lord RAYLEIGH's experiments the terminals were at 5° or 6° may explain a small part of the difference between our results, reducing his by about ·00024, it cannot possibly account for the whole, amounting as it does to ·00060, and we must look in some other direction for the explanation.

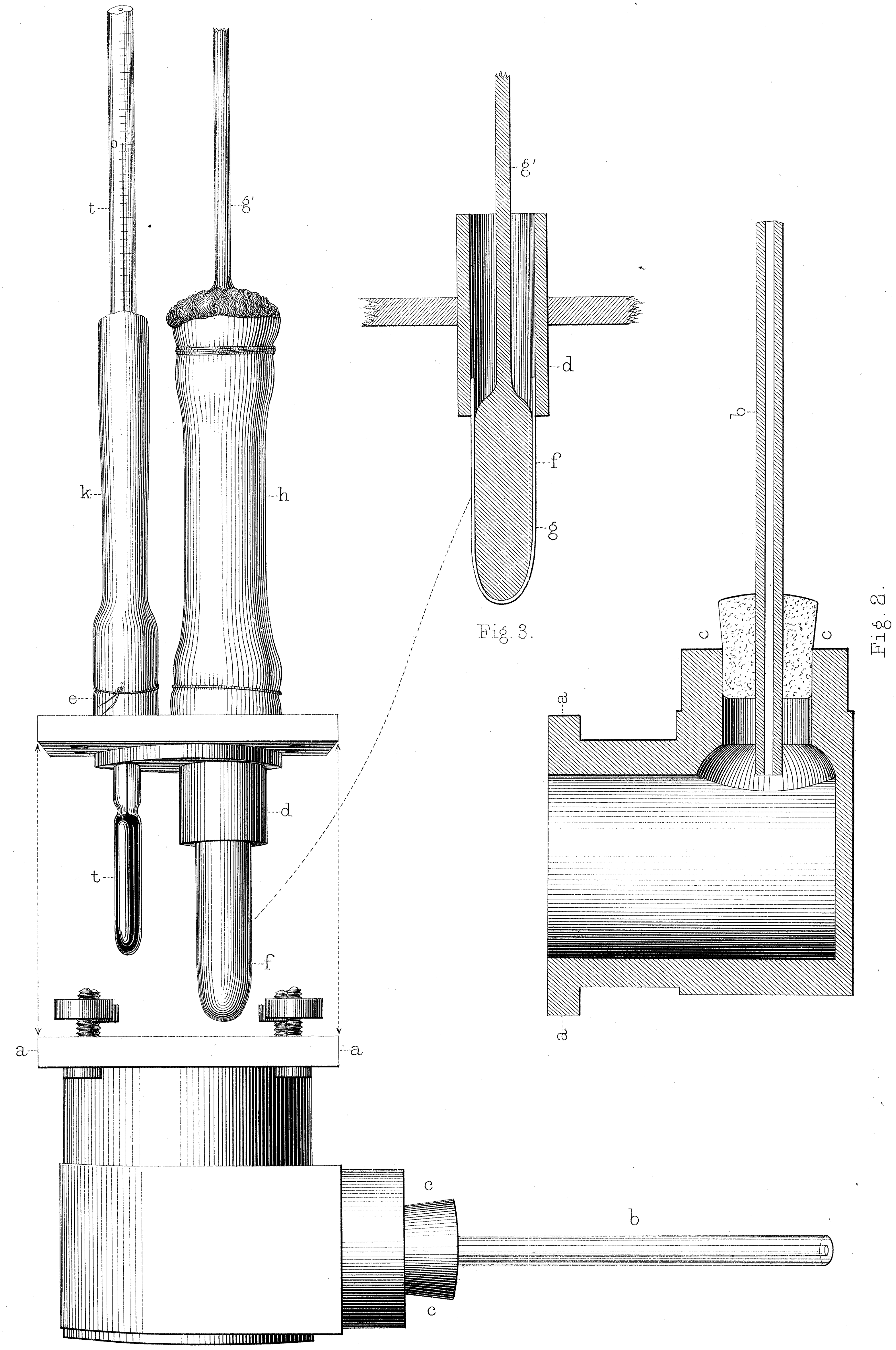

Fig. 1.

Fig. 3.

Fig. 2.

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